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Derivation of the Mechanical-energy Balance: A Plea for Simplicity

The equation

$$-W'_{sh} = \Delta \text{K.E.} + \Delta Z + \Sigma F + \int v\, dp$$

is an extremely useful one in chemical engineering and one which is derived in a variety of ways in most text books. To be consistent with requirements made of contributors, one should define the terms:

$-W'_{sh}$ = mechanical component of work delivered by a shaft machine to the substance
ΔK.E. = change in kinetic energy
ΔZ = change in potential energy
ΣF = friction
v = specific volume
p = pressure

The writer, as a teacher of thermodynamics, must take exception to the ways in which this equation is derived in several chemical engineering texts. It is not necessary to mention specific books, but a few points are worthy of attention.

It seems highly questionable to introduce the first law of thermodynamics in the batch form, i.e. $\Delta e = q - w$, in this equation, which applies only to flow processes. The idea of the gas doing work on itself, a term often met in this derivation, is difficult to understand and to impart to students when the very nature of work is the transfer of energy to or from the outside under certain potentials. It appears very dubious to derive this equation through any use of the entropy, a second-law concept, when this equation is an energy balance, a first-law concept. The use of the famous Clausius inequality

$$dS \geqq \frac{dq}{T}$$

is opposed on the score that the equation with the unequal sign has no meaning. The equation

$$dS = \frac{dq_{rev}}{T}$$

is definitional for dS. The equation with the unequal sign requires for an irreversible process the knowledge of dq, which is generally not available. Derivations which involve a differential approach and introduce such a term as dW'_{sh} must be criticized because W'_{sh} is inherently an integral and not a differential quantity.

One should go back to the derivation of Bernoulli from which there results the equation

$$d(\text{K.E.}) + dZ + v\, dp = 0$$

This is arrived at as a simple consequence of a force balance, i.e., that the pressure force acting on a given mass elevates or accelerates it. The only assumption made is that shearing forces are absent. In reality, of course, such shearing forces do exist, and these invalidate the equation. We merely make it work by adding a term for friction so that it becomes after integration

$$\Delta \text{K.E.} + \Delta Z + \int v\, dp + \Sigma F = 0$$

and thus a definition of friction. For example, when ΔK.E. + ΔZ is zero, this is exactly the equation with which friction is calculated from measured data.

Only one step remains. If mechanical energy in the form of work is added to this system in the amount $-W'_{sh}$ then equality to zero no longer holds, but rather

$$\Delta \text{K.E.} + \Delta Z + \int v\, dp + \Sigma F = -W'_{sh}$$

It is believed that this derivation is simpler and a closer approach to the truth of the matter than those usually offered.

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